$$\int \frac{dx}{x^2-6x+18} = \int \frac{dx}{(x-3)^2+9} = \int \frac{d(x-3)}{(x-3)^2+3^2} = \frac{1}{3}\,\mathrm{arctg}\left(\frac{x-3}{3}\right)+C$$

$$\int \frac{dx}{x^2-10x+21} = \int \frac{dx}{(x-5)^2-4} = \int \frac{d(x-5)}{(x-5)^2-(2)^2} = \frac{1}{2}\,\mathrm{arctg}\left(\frac{x-5}{2}\right)+C$$

Šis bija tas grūtais:

$$\int \frac{x^4+1}{x^3-x^2+x-1}\,dx =$$

$$(1x^4+0x^3+0x^2+0x+1) : (x^3-x^2+x-1) = x+1+\frac{2}{x^3-x^2+x-1}$$

$$\begin{array}{l} 1x^4-1x^3+1x^2-1x \\ \hline 1x^3-1x^2+1x+1 \\ 1x^3-1x^2+1x-1 \\ \hline 2 \end{array}$$

$$= \int\left(x+1+\frac{2}{x^3-x^2+x-1}\right)dx = \int x\,dx + \int dx + \int \frac{2}{(x^2+1)(x-1)} = \frac{x^2}{2}+x+C+\int\frac{2}{(x^2+1)(x-1)} =$$

$$\frac{2}{(x^2+1)(x-1)} = \frac{A}{x-1}^{(\cdot(x^2+1)} + \frac{Mx+N}{x^2+1}^{(\cdot(x+1)}$$

$$\frac{2}{(x^2+1)(x-1)} = \frac{A(x^2+1)+(Mx+N)(x-1)}{(x^2+1)(x-1)}$$

$$2 = A(x^2+1)+(Mx+N)(x-1)$$

1) $x-1=0$; $x=1$:

$2 = A(1+1)+(M+N)\cdot 0$

$2 = 2A \;|:2$

$A = 1$

$\boxed{A=1 \quad M=-1 \quad N=1}$

2) $x=0$:

$2 = A(0+1)+(N)(-1)$

$2 = A-N$

$2 = 1-N$

$-1 = N$

3) $x=-1$:

$2 = A(1+1)+(-M+N)(-1-1)$

$2 = 2A-2(-M+N) \;|:2$

$1 = A+M-N$

$1 = 1+M+1$

$M = -1$

$$\frac{x^2}{2}+x+C+\int\left(\frac{1}{x-1}-\frac{x+1}{x^2+1}\right)dx = \frac{x^2}{2}+x+C+\int\frac{dx}{x-1}-\int\frac{x+1}{x^2+1}dx =$$

$$\frac{x^2}{2}+x+C+\int\frac{dx}{x-1}-\int\frac{x+1}{x^2+1}dx = \frac{x^2}{2}+x+C+\int\frac{d(x-1)}{x-1}-\int\frac{x}{x^2+1}dx-\int\frac{1}{x^2+1}dx =$$

$$\frac{x^2}{2}+x+C+\ln|x-1|-\frac{1}{2}\int\frac{d(x^2)}{x^2+1}-\int\frac{dx}{x^2+1} = \frac{x^2}{2}+x+C+\ln|x-1|-\frac{1}{2}\int\frac{d(x^2+1)}{x^2+1}-\mathrm{arctg}\,x =$$

$$\frac{x^2}{2}+x+\ln|x-1|-\frac{\ln(x^2+1)}{2}-\mathrm{arctg}\,x+C$$

$$\int_0^2 \frac{dx}{\sqrt{5-x^2}} = \arcsin\frac{x}{\sqrt5}\Big|_0^2 = \arcsin\frac{2}{\sqrt5}-\arcsin\frac{0}{\sqrt5} = \arcsin\left(\frac{2\sqrt5}{5}\right)$$

$$\int_a^b u\,dv = uv\Big|_a^b-\int_a^b v\,du$$

$$\int_1^e \underbrace{x}_{dv}\underbrace{\ln x}_{u}\underbrace{dx}_{dv} = \left|\begin{array}{l} u=\ln x \\ du=u'dx=\frac{dx}{x} \\ dv=x\,dx \\ v=\int x\,dx=\frac{x^2}{2}\end{array}\right| = \frac{x^2\ln x}{2}\Big|_1^e-\int_1^e\frac{1}{2}x^2\frac{dx}{x} =$$

$$\frac{x^2\ln x}{2}\Big|_1^e - \frac{1}{2}\int_1^e x\,dx = \frac{x^2\ln x}{2}\Big|_1^e - \frac{1}{2}\left(\frac{x^2}{2}\Big|_1^e\right) = \frac{e^2}{2} - 0 - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1}{2}\right) = \frac{2e^2}{4} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2+1}{4}$$

$$\int_0^{\pi} x\sin x\,dx = \left| \begin{array}{c} u = x \\ du = dx \\ dv = \sin x\,dx \\ v = \int \sin x\,dx = -\cos x \end{array} \right| = -x\cos x\Big|_0^{\pi} - \int_0^{\pi} -\cos x\,dx = -\pi(-1) - 0 + \sin x\Big|_0^{\pi} = \pi + 0 + 0 = \pi$$

$$\int_1^4 \frac{\sqrt{x}}{1+x}\,dx = \left| \begin{array}{l} \sqrt{x} = t \quad x = t^2 \\ dx = 2t\,dt \\ x = 1 \Rightarrow t = 1 \\ x = 4 \Rightarrow t = 2 \end{array} \right| = \int_1^2 \frac{t}{1+t^2}\,2t\,dt = 2\int_1^2 \frac{t^2}{t^2+1}\,dt = 2\int_1^2 \frac{t^2+1-1}{t^2+1}\,dt =$$

$$2\int_1^2 \frac{t^2+1-1}{t^2+1}\,dt = 2\left(\int_1^2 \frac{\cancel{t^2+1}}{\cancel{t^2+1}}\,dt - \int_1^2 \frac{dt}{t^2+1}\right) = 2\left(\int_1^2 dt - \operatorname{arctg} t\Big|_1^2\right) = 2(2 - 1 - \operatorname{arctg}2 + \operatorname{arctg}1) =$$

$$2\left(1 - \operatorname{arctg}2 + \frac{\pi}{4}\right) = 2 - 2\operatorname{arctg}2 + \frac{\pi}{2}$$